Mr. Michael T. Davis WLPCS Calculus	Practice Final Exam May 15, 2018

Name: Davis / Solutions / Answers

CHAPTER 1

1. Write an equation of the line containing the points $(5,1)$ & $(5,8)$

$m = \frac{7}{0}$ = undefined equation $x = 5$

2. A line has equation $5x - 2y = -20$. Write an equation of a 2nd line that contains the point $(10,1)$ and is perpendicular to the 1st line.

$-2y = -5x - 20$
$y = \frac{5}{2}x + 10$

$y = \frac{-2}{5}x + b$
$1 = \frac{-2}{5}(10) + b$
$1 = -4 + b$
$5 = b$
$y = \frac{-2}{5}x + 5$

3. Identify the domain and range of the function $p(x) = x^2 - 4x$ $= x(x-4)$

$(0,0)$ & $(4,0)$
axis $x = 2$
$p(2) = 4 - 8 = -4$

D: $(-\infty, \infty)$ R: $[-4, \infty)$

4. Identify the domain and range of the function $w(x) = \frac{1}{x-4}$

$x \neq 4$
$y \neq 0$

D: $(-\infty, 4) \cup (4, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

5. Identify the domain and range of the function $g(x) = \sqrt{16 - x^2}$

D: $-4 \leq x \leq 4$ R: $0 \leq y \leq 4$

6. Identify the domain and range of the function $k(x) = 2\sin(3x)$

D: $(-\infty, \infty)$ R: $[-2, 2]$

7. Identify the domain and range of the function $f(x) = \ln x$

D: $x > 0$ R: $(-\infty, \infty)$

8. Identify the domain and range of the function $T(x) = \frac{1}{\sqrt{4-x}}$

D: $(-\infty, 4)$ R: $(0, \infty)$

9. The number of bacteria in a petri dish culture after t hours is given by the function $B(t) = 100e^{0.693t}$

   a. What is the initial number of bacteria present?

   100 bacteria

   b. How many bacteria are present after 6 hours?

   $B(6) = 100\, e^{0.693(6)} \approx 6{,}394$ bacteria

   c. Approximately when will the number of bacteria be 350?

   $350 = 100\, e^{0.693t}$
   $3.5 = e^{0.693t}$
   $\ln 3.5 = \ln e^{0.693t}$
   $\ln 3.5 = 0.693t$
   $\frac{\ln 3.5}{0.693} \approx t$
   $t \approx 1.808$ hours

10. Determine the zero of the function $f(x)=16-5^x$. Give the zero accurate to three decimal places.

$0=16-5^x \quad 5^x=16 \quad \ln 5^x=\ln 16 \quad x\ln 5=\ln 16 \quad x=\frac{\ln 16}{\ln 5}$

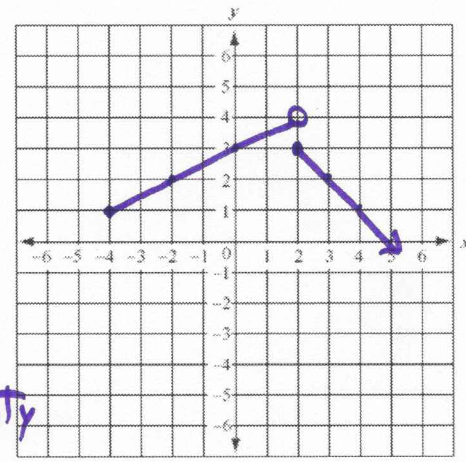
$x\approx$

11. Determine a simplified expression for $f(g(x))$ and the domain of $f(g(x))$ given the functions $f(x)=1-x^2$ and $g(x)=\sqrt{x-2}$

$f(\sqrt{x-2})=1-(\sqrt{x-2})^2=1-x+2=3-x$ Domain: $x\geq 2$

12. Given the piecewise function $f(x)=\begin{cases}\frac{1}{2}x+3 \text{ if } -4\leq x<2\\ -x+5 \text{ if } x\geq 2\end{cases}$:

a. Draw a graph of the function.



b. Determine the value of $f(2)$.

$f(2)=-2+5=3$

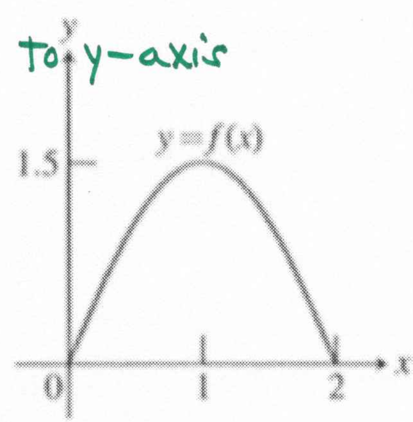
c. Determine if the function is continuous at $x=2$.

$f(x)$ is not continuous at $x=2$ since there is a jump discontinuity

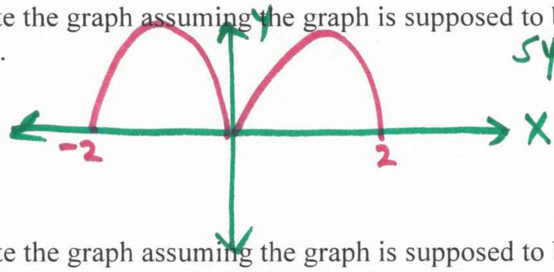
d. Determine the domain of the function.

D: $x\geq -4$

13. A portion of a graph of a function defined on the interval $[-2,2]$ is shown.

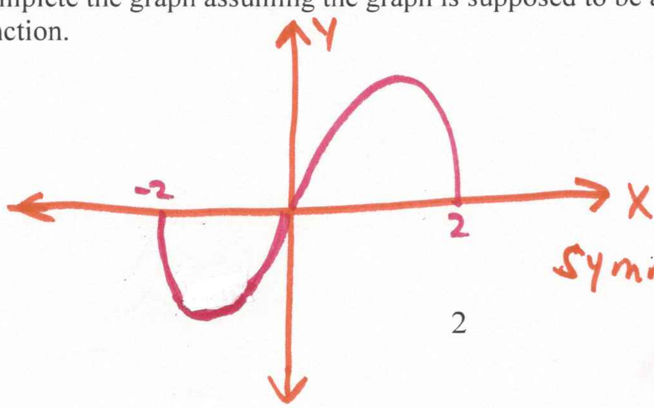


a. Complete the graph assuming the graph is supposed to be an even function.



Symmetry to y-axis

b. Complete the graph assuming the graph is supposed to be an odd function.



Symmetry to origin

$14.739 \approx t$ | $-10.217 \approx t\ln\left(\frac{1}{2}\right)$ | $-0.511 \approx \frac{t}{20}\ln\left(\frac{1}{2}\right)$

14. Suppose the half-life of a certain radioactive substance is 20 days and there are 5 grams present initially. When will there be only 3 grams of the substance remaining?

$A(t) = 5\left(\frac{1}{2}\right)^{\frac{t}{20}}$ $\quad 3 = 5\left(\frac{1}{2}\right)^{t/20}$ $\quad \frac{3}{5} = \left(\frac{1}{2}\right)^{t/20}$ $\quad \ln\left(\frac{3}{5}\right) = \ln\left(\frac{1}{2}\right)^{t/20}$

15. Determine the future value of \$4,000 invested for twelve years in an interest bearing savings account that earns 6.25% compounded continuously.

$A(t) = 4000\, e^{0.0625t}$ $\quad A(12) = 4000\, e^{0.0625(12)} \approx \$8{,}468$

16. If $g(x) = 2x - 6$, then determine $g^{-1}(x)$.

$y = 2x - 6$ $\quad x = 2y - 6$ $\quad 2y = x + 6$ $\quad y = \frac{1}{2}x + 3$ $\quad g^{-1}(x) = \frac{1}{2}x + 3$

17. If $k(x) = \ln(x)$, then determine $k^{-1}(x)$.

$y = \ln x$ $\quad x = \ln y$ $\quad y = e^x$ $\quad k^{-1}(x) = e^x$

18. Determine if the function $g(x) = x^2 - 1$ is a one-to-one function. Answer with "yes" or "no" and explain why ore why not.

$g(3) = 3^2 - 1 = 8$ $\quad g(-3) = (-3)^2 - 1 = 8$

The function $g(x)$ is not one to one since there are two x's with the same y.

19. Determine if the function $p(x) = \dfrac{1}{x-3}$ is a one-to-one function. Answer with "yes" or "no" and explain why ore why not.

$p(x)$ is one to one since there corresponds exactly one y for each x and there corresponds exactly one x for each y.

20. $y = f(x)$ is a one-to-one function such that $f\left(\frac{1}{2}\right) = -4$. Determine the value of $f^{-1}(-4)$ $= \frac{1}{2}$

21. Solve the equation $6^t = 50$ for t.

$\ln 6^t = \ln 50$
$t \ln 6 = \ln 50$
$t = \dfrac{\ln 50}{\ln 6}$
$t \approx 2.183$

22. Solve the equation $\log_m 64 = 3$ for m.

$m^3 = 64$ $\quad m = 4$ $\quad$ m cannot be negative

23. Solve the equation $\ln k = -1$ for k.

$e^{-1} = k$ $\quad k = \frac{1}{e}$

24. Solve the equation $\log t + \log 20 = 2$ for t.

$\log(20t) = 2$ $\quad 10^2 = 20t$ $\quad 100 = 20t$ $\quad t = 5$ $\quad \{5\}$

25. True or False: $\log_n x - \log_n y = \log_n\left(\dfrac{x}{y}\right)$

$\log_n\left(\frac{x}{y}\right) = \log_n\left(\frac{x}{y}\right)$

26. True or False: $\log_n x - \log_n y = \dfrac{\log_n x}{\log_n y}$ $\log_n x - \log_n y = \log_n\left(\frac{x}{y}\right) \neq \dfrac{\log_n x}{\log_n y}$

27. True or False: $\ln x = \dfrac{\log x}{\log a}$ change of base formula $\dfrac{\log x}{\log a} = \log_a x \neq \ln x$

28. Evaluate the expression $a^{\log_a 7}$ $= 7$

29. Evaluate the expression $\ln e^4$. $= 4$

30. True or False: $g(x) = \sin x$ is an even function.

31. True or False: $k(x) = \cos x$ is an even function.

32. Determine the value of each trigonometric expression:

a. $\sin\dfrac{\pi}{6} = \dfrac{1}{2}$

b. $\tan\dfrac{\pi}{4} = 1$

c. $\cos\dfrac{\pi}{3} = \dfrac{1}{2}$

d. $\tan\dfrac{\pi}{2} =$ undefined

e. $\sec\dfrac{5\pi}{6} = \dfrac{1}{\cos\frac{5\pi}{6}} = \dfrac{1}{-\frac{\sqrt{3}}{2}} = \dfrac{-2}{\sqrt{3}} = \dfrac{-2\sqrt{3}}{3}$

f. $\cos\pi = -1$

g. $\cot\dfrac{4\pi}{3} = \dfrac{1}{\tan\frac{4\pi}{3}} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

h. $\sin\dfrac{3\pi}{2} = -1$

i. $\csc\dfrac{5\pi}{3} = \dfrac{1}{\sin\frac{5\pi}{3}} = \dfrac{1}{-\frac{\sqrt{3}}{2}} = \dfrac{-2}{\sqrt{3}} = \dfrac{-2\sqrt{3}}{3}$

j. $\cot\dfrac{11\pi}{6} = \dfrac{1}{\tan\frac{11\pi}{6}} = \dfrac{1}{-\frac{\sqrt{3}}{3}} = \dfrac{-3}{\sqrt{3}} = -\sqrt{3}$

33. Determine the value of each inverse trigonometric expression:

a. $\sin^{-1}(0.5)$ $= \frac{\pi}{6}$

b. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{\pi}{4}$

c. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\pi}{6}$

d. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\pi}{3}$

e. $\sin^{-1}(1)$ $= \frac{\pi}{2}$

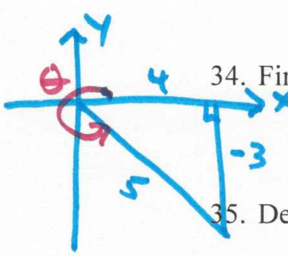
f. $\cos^{-1}(-1)$ $= \pi$

g. $\cos^{-1}(-0.5)$ $= \frac{2\pi}{3}$

h. $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ $= -\frac{\pi}{3}$

i. $\tan^{-1}(-\sqrt{3})$ $= -\frac{\pi}{3}$

j. $\sin^{-1}(0)$ $= 0$



34. Find the five other trigonometric ratios of θ if $\sin\theta = \frac{-3}{5}$, $\tan\theta < 0$ and $0 \le \theta < 2\pi$.

$\cos\theta = \frac{4}{5}$ $\csc\theta = \frac{-5}{3}$ $\cot\theta = \frac{-4}{3}$

$\sec\theta = \frac{5}{4}$ $\tan\theta = \frac{-3}{4}$

35. Determine the amplitude, period, phase shift, vertical shift and range of $f(x) = -3\sin(2x+\pi)+1$

3 $\frac{2\pi}{2}$ $\frac{\pi}{2}$ left up 1 $-2 \le y \le 4$ $f(x) = 3\sin\left(2\left(x+\frac{\pi}{2}\right)\right)+1$

π

CHAPTER 2

1. Find the average rate of change of $p(x) = 3x^2 - x$ over the interval $[1,3]$.

$\frac{3(3)^2 - 3 - (3(1)^2 - 1)}{3-1} = \frac{24-2}{2} = \frac{22}{2} = 11$

2. Find the instantaneous rate of change of $p(x) = 3x^2 - x$ at $x = 2$.

$p'(x) = 6x - 1$ $p'(2) = 6(2) - 1 = 11$

3. Write an extended function for $g(y) = \frac{y^2 - 3y + 2}{y^2 - 4}$ that is continuous at $y = 2$.

$g(y) = \frac{(y-1)(y-2)}{(y+2)(y-2)}$

has hole at $y=2$

removeable discontinuity at $y=2$

$f(y) = \frac{y-1}{y+2}$

extended function

does not have a hole at $y=2$

4. Find a left end behavior model for $f\left(x\right)=\sin x+e^{x}$.

5. Evaluate each limit

a. $\lim\limits_{x\to 2}\left(3x+1\right)=$ 7

b. $\lim\limits_{k\to 2}\left(3k\right)=$ 6

c. $\lim\limits_{x\to\frac{\pi}{6}}\sin x=$ $\sin\frac{\pi}{6}=\frac{1}{2}$

d. $\lim\limits_{x\to\frac{\pi}{2}^{+}}\tan x=$ $-\infty$ graph on your calculator

e. $\lim\limits_{x\to 5}\dfrac{x-5}{x+1}=$ $\frac{0}{6}=0$

f. $\lim\limits_{x\to 5^{+}}\dfrac{x+1}{x-5}=$ ∞ graph on your calculator

g. $\lim\limits_{x\to 5^{-}}\dfrac{x+1}{x-5}=$ $-\infty$ Graph on your Calculator

h. $\lim\limits_{x\to 5}\dfrac{x+1}{x-5}=$ Undefined since $\lim\limits_{x\to 5^{-}}\frac{x+1}{x-5}\neq\lim\limits_{x\to 5^{+}}\frac{x+1}{x-5}$

i. $\lim\limits_{x\to\infty}\dfrac{x+1}{x-5}=$ $\frac{1}{1}=1$

j. $\lim\limits_{x\to 3}\dfrac{x^{2}-9}{x-3}=$ $\lim\limits_{x\to 3}\frac{(x-3)(x+3)}{x-3}=6$

k. $\lim\limits_{t\to 0}\dfrac{\sin t}{t}=$ 1 graph on your calculator

l. $\lim\limits_{t\to\infty}\dfrac{\sin t}{t}=$ 0 graph on your calculator

6. Given $f\left(t\right)=\begin{cases}2t+4 \text{ if } t\leq -1\\ -3t-2 \text{ if } t>-1\end{cases}$, $\lim\limits_{t\to -1^{+}}f\left(t\right)=$ $-3(-1)-2=3-2=1$

7. Given $g\left(x\right)=\begin{cases}5-x \text{ if } x<2\\ \frac{1}{2}x+2 \text{ if } x\geq 2\end{cases}$, $\lim\limits_{x\to 2}g\left(x\right)=$ 3

$5-2=3$

$\frac{1}{2}(2)+2=3$

8. A small rock is dropped from the top of a tall cliff. What is the average speed in feet per second of the falling rock from time $t=2$ seconds to $t=3$ seconds, i.e., during the third second of its fall? The equation $y=16t^{2}$ defines the number of feet the object falls during the first t seconds.

Average speed $=\dfrac{16(3)^2-16(2)^2}{3-2}=\dfrac{16(9-4)}{1}=16(5)=80\,\frac{\text{ft}}{\text{sec}}$

9. Find all values of k for which the function $f(x)=\begin{cases} x+4 & \text{if } x\le 1 \\ x^2-2x+k & \text{if } x>1 \end{cases}$ is continuous.

$1+4=(1)^2-2(1)+K$

$5=-1+K$ $\quad\quad$ $K=6$

10. For each rational function, (1) determine an end behavior model, (2) evaluate $\lim_{x\to\infty} f(x)$, and (3) determine equations for any horizontal or vertical asymptotes.

→ EBM

a. $f(x)=\dfrac{6x^3-1}{3x^3+4x}$

EBM $g(x)=2$

$\lim_{x\to\infty} f(x)=2$ $\quad$ H.A. $y=2$ $\quad$ V.A. $x=0$

b. $f(x)=\dfrac{x^4-3}{2x^3-8x}$

EBM $g(x)=\frac{1}{2}x$

$\lim_{x\to\infty} f(x)=\infty$ $\quad$ H.A. None $\quad$ V.A. $x=0$, $x=2$, $x=-2$

c. $f(x)=\dfrac{4x^2+x}{x^3+8}$

EBM $g(x)=\frac{4}{x}$

$\lim_{x\to\infty} f(x)=0$ $\quad$ H.A. $y=0$ $\quad$ V.A. $x=-2$

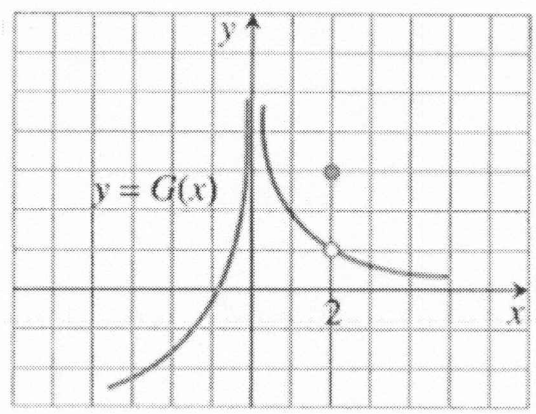
11. Determine the type of discontinuity for each function.

a. $f(x)=\dfrac{x^2-81}{x-9}$ Removeable Distont.

b. $g(x)=\dfrac{x-9}{x^2-81}$ Asymptotic Discont.

c. $p(x)=\dfrac{|x-2|}{x-2}$ Jump Discont.

12. Given the graph of $y=G(x)$, determine each of the following:



a. $G(2)=$ 3

b. $\lim_{x\to 2} G(x)=$ 1

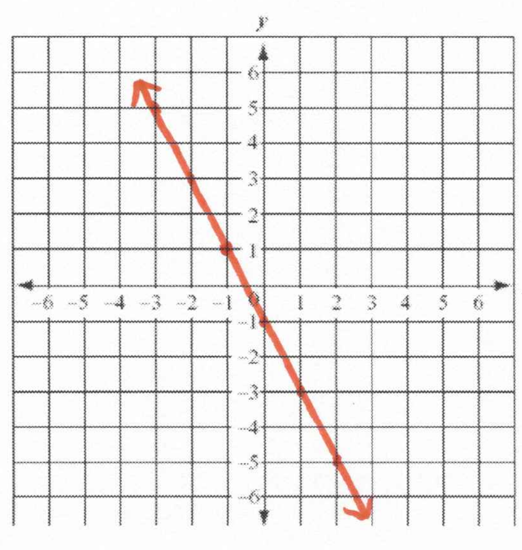
c. $\lim_{x\to 0^+} G(x)=\infty$

CHAPTER 3

1. Sketch a graph of a continuous function $y=f(x)$ that has the following properties:

i. $f(-1)=1$,

ii. $f'(x)=-2$.



$$\lim_{h\to 0} \frac{6(2+h)^2-5(2+h)-(6(2)^2-5(2))}{h} = \lim_{h\to 0} \frac{6(4+4h+h^2)+10-5h-24+10}{h}$$

$$= \lim_{h\to 0} \frac{6h^2+19h}{h} = \lim_{h\to 0}(6h+19) = 19$$

2. The function $f(x)=\begin{cases} x^2 \text{ if } x>2 \\ 4x \text{ if } x\le 2 \end{cases}$ has left-hand and right-hand derivatives at $x=2$. Does $f(x)$ have a derivative at $x=2$? Explain why or why not.

$f'(x) = \begin{cases} 2x & \text{if } x>2 \\ 4 & \text{if } x\le 2 \end{cases}$

$\lim_{x\to 2^+} f'(x) = 4$ $\lim_{x\to 2^-} f'(x) = 4$ f(x) has a derivative at x = 2 since the left hand and right hand derivative are equal

3. If $f(x)=6x^2-5x$, then $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h} =$

See above

4. If $f(-3)=4$ and $f'(-3)=-1$, write an equation of the line tangent to the graph of $y=f(x)$ at the point where $x=-3$.

$y-4=-(x+3)$

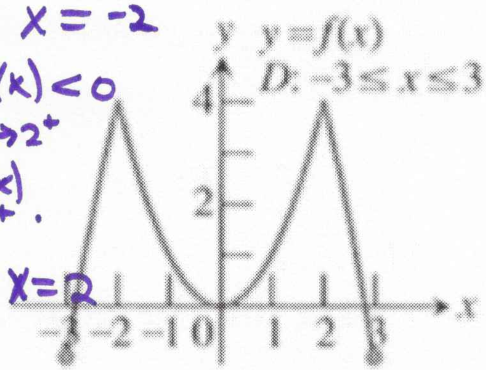
5. Determine the value(s) of x on the interval $[0,2\pi]$ for which the function $f(x)=\tan x$ is not differentiable. Explain why.

f(x) is not differentiable at $x=\frac{\pi}{2}$ and at $x=\frac{3\pi}{2}$ since $f(\frac{\pi}{2})$ = undefined and $f(\frac{3\pi}{2})$ = undefined

6. The graph of a function $y=f(x)$ is shown below. State the values of x for which the function is not differentiable. Explain why.

f(x) is not differentiable at $x=-2$ since $\lim_{x\to -2^-} f'(x) > 0$ and $\lim_{x\to 2^+} f'(x) < 0$ therefore $\lim_{x\to 2^-} f'(x) \ne \lim_{x\to -2^+} f'(x)$.

f(x) is not differentiable at $x=2$ for the same reason.



7. Find $f'(x)$ for the function $f(x)=\frac{x^3}{9}+\frac{x^2}{4}-\frac{x}{2}+\frac{1}{3}$

$f'(x) = \frac{x^2}{3}+\frac{x}{2}-\frac{1}{2}$

8. Find $\frac{dy}{dx}$ for the function $y=2\sqrt{x}+3\sqrt[3]{x}$

$\frac{dy}{dx} = x^{-\frac{1}{2}} + x^{-2/3}$

$y = 2x^{\frac{1}{2}}+3x^{1/3}$

9. Find y' for the function $y=\frac{2}{x}-\frac{4}{x^2}$

$y' = -2x^{-2}+8x^{-3}$

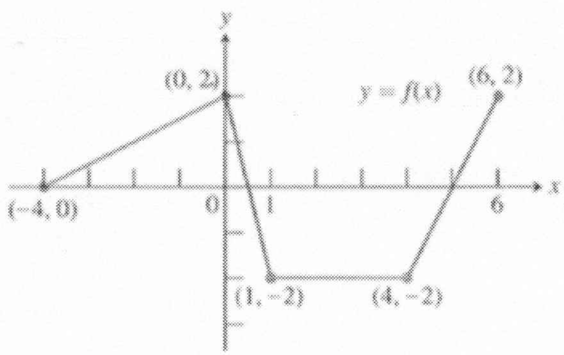
$y = 2x^{-1}-4x^{-2}$

10. Find $\frac{dy}{dx}$ for the function $y=\frac{x^3+1}{\sqrt{x}}$ $\quad \frac{dy}{dx} = \frac{(3x^2)(\sqrt{x})-(x^3+1)\cdot\frac{1}{2}x^{-\frac{1}{2}}}{x}$

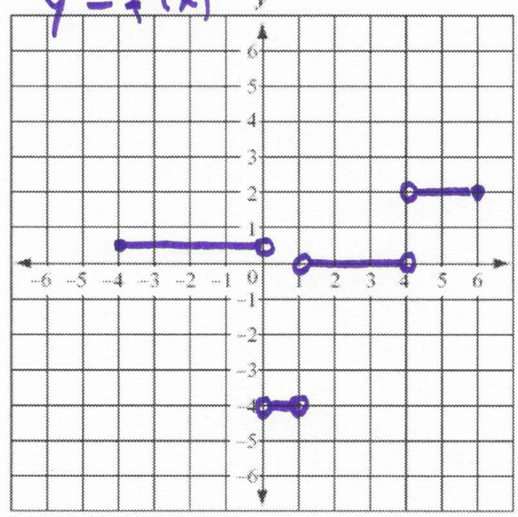
11. Find $f''(x)$ for the function $f(x)=(x-1)^5$

$f'(x)=5(x-1)^4 \quad f''(x)=20(x-1)^3$

12. The graph of the function $y=f(x)$ shown here is made of the line segments joined end to end. Graph the function's derivative function.



$y=f'(x)$



13. Particle Motion. A particle moves along a real number line (left and right) so that its position at any time $t \geq 0$ is given by the function $s(t)=t^3-5t^2+3t-2$ where s is measured in meters and t is measured in seconds. Positive velocity implies movement to the right.

a. Determine the particles displacement from $t=1$ and $t=3$.

$s(3)-s(1) = 3^3-5(3)^2+3(3)-2-(1^3-5(1)^2+3(1)-2) = -8\text{ m}$

b. Determine the average velocity from $t=1$ and $t=3$.

$\frac{s(3)-s(1)}{3-1} = \frac{-8}{2} = -4 \text{ m/sec}$

c. Find the instantaneous velocity at any time t.

$s'(t)=v(t)=3t^2-10t+3$

d. At what time(s) does the particle change direction? Justify your answer.

$0=3t^2-10t+3 \quad 0=(3t-1)(t-3) \quad t=\frac{1}{3} \quad t=3$

s': $++$ | $\frac{1}{3}$ | $---$ | 3 | $++$

e. Is the particle moving forward or backward at time $t=4$ sec ? Justify your answer.

The particle is changing direction at $t=\frac{1}{3}$ and $t=3$.
The particle moves forward on $(-\infty,\frac{1}{3})\cup(3,\infty)$ since $s'>0$
The particle moves backward on $(\frac{1}{3},3)$ since $s'<0$

$s(t)$ changes direction at $t=\frac{1}{3}$ and $t=3$ since $s'(t)$ changes sign

14. Projectile Motion. A rocket propelled vertically upward from the surface of the Earth at an initial velocity of $39.2\,m/\sec$ reaches a height of $h(t) = 39.2t - 4.9t^2$ meters in t seconds.

a. Find the rock's velocity at time $t = 5$?

$V(t) = 39.2 - 9.8t \qquad V(5) = 39.2 - 9.8(5) = -9.8\,m/sec$

b. Find the rocket's acceleration at time $t = 5$?

$a(t) = -9.8\,m/s^2$

c. How long does it take the rocket to reach its highest point? Justify your answer.

$V(t) = 0 \qquad 0 = 39.2 - 9.8t \qquad t = 4$

+++ 4 − − − V

It take 4 sec to reach its highest point since V(t) changes from Pos. to Neg.

d. How high does the rocket go?

$h(4) = 3.92(4) - 4.9(5)^2 = -106.82m$

15. Write an equation for the line tangent to the graph of $f(x) = -2x^3 + 10x + 5$ at $x = 2$.

$f(2) = -2(2)^3 + 10(2) + 5 = -16 + 20 + 5 = 9$

$f'(x) = -6x^2 + 10 \qquad f'(2) = -6(2)^2 + 10 = -14 \qquad y - 9 = -14(x - 2)$

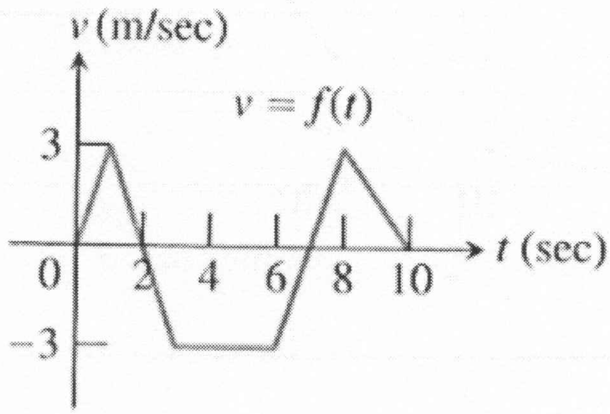
16. Determine an equation of the line normal to the graph of $f(x) = \cos x$ at $x = \frac{\pi}{2}$.

$f(\frac{\pi}{2}) = \cos\frac{\pi}{2} = 0 \qquad f'(x) = -\sin x \qquad f'(\frac{\pi}{2}) = -\sin\frac{\pi}{2} = -1$

Normal slope $m = 1$

$y - 0 = 1(x - \frac{\pi}{2}) \qquad y = x - \frac{\pi}{2}$

17. Particle Motion. The accompanying figure shows the velocity $V = f(t)$ of a particle moving on a coordinate line.



a. When does the particle move forward? Justify your answer.

Particle moves forward on (0,2) and (7,10) since $V(t) > 0$

b. When does the particle speed up? Justify your answer.

Particle speeds up on $(0,1) \cup (2,3) \cup (7,8)$ since V(t) and a(t) have the same sign.

c. When is the particle's acceleration negative? Justify your answer.

acceleration is negative on $(1,3) \cup (8,10)$ since $V'(t) < 0$

d. When is the particle's acceleration zero? Justify your answer.

CHAPTER 4

acceleration is zero on (3,6) since $V'(t) = a(t) = 0$

1. Find $f'(x)$ for the function $f(x)=(3x+1)^6$

$f'(x) = 6(3x+1)^5(3)$

2. Find $\frac{dy}{dt}$ for the function $y=\sqrt{10t}$

$\frac{dy}{dt} = \frac{1}{2}(10t)^{-\frac{1}{2}}(10) = \frac{5}{\sqrt{10t}}$

3. Find y' for the function $y=x\sqrt{\sin x}$

$y' = \sqrt{\sin x} + x \cdot \frac{1}{2}(\sin x)^{-\frac{1}{2}} \cdot (+\cos x) = \sqrt{\sin x} + \frac{x\cos x}{2\sqrt{\sin x}}$

4. Find $\frac{dy}{dt}$ given $y=\frac{\tan(1-3t)}{2t}$

$\frac{dy}{dt} = \frac{-3\sec^2(1-3t)(2t) - 2\tan(1-3t)}{4t^2}$

5. Find $f'(x)$ for the function $f(x)=(x^2+3x)^{-2}$

$f'(x) = -2(x^2+3x)^{-3}(2x+3) = \frac{-4x-6}{(x^2+3x)^3}$

6. Find $\frac{dy}{dt}$ for the function $y=\csc^2(\pi t)$

$\frac{dy}{dt} = 2\csc(\pi t)(-\csc(\pi t)\cot(\pi t))(\pi)$

7. Find y' for the function $y=\left(\cos x+\frac{1}{x}\right)^3$

$y' = 3\left(\cos x + \frac{1}{x}\right)^2\left(-\sin x - \frac{1}{x^2}\right)$

8. Find $f'(\theta)$ for the function $y=\sec(3\theta)\tan(4\theta)$

$f'(\theta) = 3\sec(3\theta)\tan(3\theta)\tan(4\theta) + \sec(3\theta) \cdot 4\sec^2(4\theta)$

9. Find y' for the function $y=\frac{2x}{\cot^3 x}$

$y' = \frac{2\cot^3 x - 2x \cdot 3\cot^2 x(-\csc^2 x)}{\cot^6 x}$

10. Find $\frac{dy}{dx}$ given $y=\sin^3(4x-1)$.

$\frac{dy}{dx} = 3\sin^2(4x-1) \cdot \cos(4x-1) \cdot 4$

11. Find $\frac{dy}{dx}$ given $y=\frac{\cos(\pi x)}{\sqrt{x}}$.

$\frac{dy}{dx} = \frac{-\pi\sin(\pi x)\sqrt{x} - \cos(\pi x) \cdot \frac{1}{2}x^{-\frac{1}{2}}}{x}$

12. Determine the value of $\frac{dy}{dx}$ given $y=\sin^{-1}(2x)$

$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$

13. Determine an expression for $\frac{dy}{dx}$ given $y=\sec(x)\tan(x)$

$\frac{dy}{dx} = \sec x \tan x \cdot \tan x + \sec x \cdot \sec^2 x = \sec x \tan^2 x + \sec^3 x$

14. If $f(x)=x\ln x$, then $f'(1)=$

$f'(x) = \ln x + x \cdot \frac{1}{x} = \ln x + 1 \quad f'(1) = \ln 1 + 1 = 0+1 = 1$

15. Find $\frac{dy}{dx}$ given $x^2+y^2=9$

$2x + 2y\frac{dy}{dx} = 0 \quad y\frac{dy}{dx} = -x \quad \frac{dy}{dx} = \frac{-x}{y}$

16. Find y' given $yx^2+y^2=x$

$y'x^2 + y \cdot 2x + 2y \cdot y' = 1 \quad y' = \frac{1-2xy}{x^2+2y}$

17. Find $\frac{dy}{dx}$ given $x=\sin y$

$1 = \cos y \cdot y' \quad y' = \frac{1}{\cos y} = \sec y$

18. Find y' given $x+\cos y = xy$

$1 - \sin y \cdot y' = y + xy' \quad 1-y = xy' + \sin y \cdot y' \quad y' = \frac{1-y}{x+\sin y}$

19. Find $\frac{dy}{dx}$ given $\tan(xy)=6+y$

$\sec^2(xy)(y+xy') = y' \quad y\sec^2(xy) + xy'\sec^2(xy) = y'$

$y' = \frac{y\sec^2(xy)}{1-x\sec^2(xy)}$

20. Find y' given $6x^2-3xy+2y^2+7x=5$

$12x - 3y - 3xy' + 4y \cdot y' + 7 = 0 \quad y' = \frac{3y-12x-7}{4y-3x}$

21. Find y' and the slope of the curve of $x^2+y^2=25$ at the point $(-3,4)$

$2x + 2y \cdot y' = 0 \quad y' = -\frac{x}{y} \quad y' = \frac{3}{4}$

22. Find y' and the slope of the curve of $(x+2)^2+(y+3)^2=25$ at the point $(1,-7)$

$2(x+2) + 2(y+3)y' = 0 \quad y' = \frac{-x-2}{y+3} \quad y' = \frac{-3}{-4} = \frac{3}{4}$

23. Given the relation $y^2-2x=x^2$ find $\frac{dy}{dx}$ and then find $\frac{d^2y}{dx^2}$

$2yy' - 2 = 2x \quad y' = \frac{2x+2}{2y} = \frac{x+1}{y} \quad y'' = \frac{y-(x+1)(y')}{y^2}$

$y'' = \frac{y-(x+1)\left(\frac{x+1}{y}\right)}{y^2}$

$y'' = \frac{y^2-(x+1)^2}{y^3}$

24. Given the relation $y^2+2y=2x+1$ find $\frac{dy}{dx}$ and then find $\frac{d^2y}{dx^2}$

$2yy' + 2y' = 2$

$y' = \frac{1}{y+1} \quad y'' = \frac{0(y+1)-1(y')}{(y+1)^2} = \frac{-1}{(y+1)^3}$

25. Find the slope of the curve of $xy^2+x=1$ at the point $\left(\frac{1}{2},1\right)$

$y^2+x\cdot 2yy'+1=0$

$y'=\frac{-1-y^2}{2xy}$ $\quad y'=\frac{-1-1}{2\left(\frac{1}{2}\right)(1)}=\frac{-2}{1}=-2$ $\quad m=-2$

26. Determine an equation for the line normal to the ellipse with equation $x^2-xy+y^2=7$ at the point $(-1,2)$.

$2x-y-xy'+2yy'=0$ $\quad y'=\frac{2-2(-1)}{2(2)-(-1)}=\frac{4}{5}$ $\quad m=\frac{-5}{4}$

$y'=\frac{y-2x}{2y-x}$

27. Find $\frac{dy}{dx}$ given $y=\sin^{-1}(x)$

$y'=\frac{1}{\sqrt{1-x^2}}$

28. Find $\frac{dy}{dx}$ given $y=\cos^{-1}(x)$

$\frac{dy}{dx}=\frac{-1}{\sqrt{1-x^2}}$

29. Find $\frac{dy}{dx}$ given $y=\tan^{-1}(x)$

$\frac{dy}{dx}=\frac{1}{1+x^2}$

30. Find $\frac{dy}{dx}$ given $y=\cot^{-1}(x)$

$\frac{dy}{dx}=\frac{-1}{1+x^2}$

31. Find $\frac{dy}{dx}$ given $y=\sec^{-1}(x)$

$\frac{dy}{dx}=\frac{1}{|x|\sqrt{x^2-1}}$

32. Find $\frac{dy}{dx}$ given $y=\csc^{-1}(x)$

$\frac{dy}{dx}=\frac{-1}{|x|\sqrt{x^2-1}}$

33. Find $\frac{dy}{dx}$ given $y=\cos^{-1}(x^2)$

$\frac{dy}{dx}=\frac{-2x}{\sqrt{1-x^4}}$

34. Find y' given $y=\sin^{-1}(t^2+t)$

$y'=\frac{2t+1}{\sqrt{1-(t^2+t)^2}}$

35. Find $\frac{dy}{dt}$ given $y=2x\tan^{-1}\left(\frac{1}{x}\right)$

$\frac{dy}{dx}=2\tan^{-1}\left(\frac{1}{x}\right)+2x\frac{-\frac{1}{x^2}}{1+\frac{1}{x^2}}$

$=2\tan^{-1}\left(\frac{1}{x}\right)-\frac{2x}{x^2+1}$

36. Find $\frac{dy}{dx}$ given $y=\cot^{-1}\left(\sqrt{2x}\right)$ $\frac{dy}{dx} = \frac{-\frac{1}{2}(2x)^{-\frac{1}{2}}}{1+2x} = \frac{-1}{2\sqrt{2x}(1+2x)}$

37. A particle moves along the x-axis so that its position in meters at any time $t \geq 0$ is given by $x(t)=\sin^{-1}(2t)$. Find the velocity of the particle at $t=\frac{1}{4}$ seconds.

$V(t) = \frac{2}{\sqrt{1-4t^2}}$ $V\left(\frac{1}{4}\right) = \frac{2}{\sqrt{1-\frac{1}{4}}} = \frac{2}{\sqrt{\frac{3}{4}}} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

38. A particle moves along the x-axis so that its position in meters at any time $t \geq 0$ is given by $x(t)=\tan^{-1}\left(\frac{t}{2}\right)$. Find the velocity of the particle at $t=1$ seconds.

$V(t) = \frac{\frac{1}{2}}{1+\frac{t^2}{4}} = \frac{2}{4+t^2}$ $V(1) = \frac{2}{5}$

39. Write an equation for the line tangent to the graph of $f(x)=\cos^{-1}x$ at the point $\left(\frac{\sqrt{3}}{2},\frac{\pi}{6}\right)$.

$f\left(\frac{\sqrt{3}}{2}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$ $f'(x) = \frac{-1}{\sqrt{1-x^2}}$ $f'\left(\frac{\sqrt{3}}{2}\right) = \frac{-1}{\sqrt{\frac{1}{4}}} = -2$

$y - \frac{\pi}{6} = -2\left(x - \frac{\sqrt{3}}{2}\right)$

40. Find $\frac{dy}{dx}$ given $y=\ln x$

$\frac{dy}{dx} = \frac{1}{x}$

41. Find $\frac{dy}{dx}$ given $y=(\ln x)^2$

$\frac{dy}{dx} = 2\ln x \cdot \frac{1}{x} = \frac{2\ln x}{x}$

42. Find $\frac{dy}{dx}$ given $y=\ln(7x)$

$\frac{dy}{dx} = \frac{1}{7x} \cdot 7 = \frac{1}{x}$

43. Find $\frac{dy}{dx}$ given $y=\log x$

$\frac{dy}{dx} = \frac{1}{x\ln 10}$

44. Find $\frac{dy}{dx}$ given $y=\log(9x)$

$\frac{dy}{dx} = \frac{9}{9x\ln 10} = \frac{1}{x\ln 10}$

45. Find $\frac{dy}{dx}$ given $y=\log(x^3-2)$

$\frac{dy}{dx} = \frac{3x^2}{(x^3-2)\ln 10}$

46. Find $\frac{dy}{dx}$ given $y=\log_5 x$

$\frac{dy}{dx} = \frac{1}{x\ln 5}$

47. Find $\frac{dy}{dx}$ given $y=e^x$

$$\frac{dy}{dx}=e^x$$

48. Find $\frac{dy}{dx}$ given $y=e^{\pi x}$

$$\frac{dy}{dx}=\pi e^{\pi x}$$

49. Find $\frac{dy}{dx}$ given $y=e^{\tan x}$

$$\frac{dy}{dx}=\sec^2 x\cdot e^{\tan x}$$

50. Find $\frac{dy}{dx}$ given $y=\left(e^x+\ln x\right)^2$

$$\frac{dy}{dx}=2\left(e^x+\ln x\right)\left(e^x+\frac{1}{x}\right)$$

51. Find $\frac{dy}{dx}$ given $y=e^x\cdot\sin^{-1}x$

$$\frac{dy}{dx}=e^x\cdot\sin^{-1}x+\frac{e^x}{\sqrt{1-x^2}}$$

52. Find $\frac{dy}{dx}$ given $y=2^x$

$$\frac{dy}{dx}=2^x\cdot\ln 2$$

53. Find $\frac{dy}{dx}$ given $y=2^{\cos x}$

$$\frac{dy}{dx}=2^{\cos x}(-\sin x)\ln 2$$

54. Find $\frac{dy}{dx}$ given $y=\sin^{-1}\left(\ln x\right)$

$$\frac{dy}{dx}=\frac{\frac{1}{x}}{\sqrt{1-(\ln x)^2}}=\frac{1}{x\sqrt{1-(\ln x)^2}}$$

55. Find y' given $y=\cos^{-1}\left(3^x\right)$

$$y'=\frac{-3^x\cdot\ln 3}{\sqrt{1-(3^x)^2}}$$

56. Find $\frac{dy}{dx}$ given $y=\tan^{-1}\left(e^{4x}\right)$

$$\frac{dy}{dx}=\frac{4e^{4x}}{1+(e^{4x})^2}=\frac{4e^{4x}}{1+e^{8x}}$$

57. Determine $\frac{dy}{dx}$ given $y=xe^{5x+1}$

$$\frac{dy}{dx}=e^{5x+1}+x\cdot 5e^{5x+1}=e^{5x+1}(1+5x)$$

58. Determine $\frac{dy}{dx}$ given $y = 3^x \ln(\cos x)$

$\frac{dy}{dx} = 3^x \cdot \ln 3 \cdot \ln(\cos x) + 3^x\left(\frac{-\sin x}{\cos x}\right)$

Chapter 5

f(x) has an abs. min. of $y=-1$ at $x=\frac{3\pi}{2}$ since $f'(x)$ changes from Neg. to Pos.

f(x) has an abs. max. of $y=1$ at $x=\frac{\pi}{2}$ since $x=\frac{\pi}{2}$ is an endpoint

f(x) has a relative max of $y=0$ at $x=2\pi$ since $x=2\pi$ is an endpoint

1. Use Calculus to determine the extreme values of the function $f(x) = \sin(x)$ on $\left[\frac{\pi}{2}, 2\pi\right]$

$f'(x) = \cos x$

$f'\left(\frac{\pi}{2}\right) = 0$ $f'\left(\frac{3\pi}{2}\right) = 0$

$- - - -$ $+ + + +$ on $\frac{\pi}{2}$, $\frac{3\pi}{2}$, 2π

$\sin\frac{\pi}{2} = 1$ $\sin 2\pi = 0$ $\sin\frac{3\pi}{2} = -1$

2. Use Calculus to determine the extreme values of the function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x - 1$

$f'(x) = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x = -2$ c.p.

$x = 1$ c.p.

$+++$ $---$ $+++$ f' at -2, 1; $y = \frac{7}{3}$, $y = -2\frac{1}{6}$

f(x) has a local max of $y = \frac{7}{3}$ at $x = -2$ since $f'(x)$ changes from Pos. to Neg. f(x) has a local min of $y = -2\frac{1}{6}$ at $x = 1$ since $f'(x)$ changes from Neg. to Pos.

3. Determine the interval(s) on which the function $f(x) = e^{x^2-6x}$ is (a) increasing and (b) decreasing.

$f'(x) = (2x-6)e^{x^2-6x}$

$0 = 2x - 6$

$x = 3$

$- - -$ $+ + +$ f' at 3

f(x) is dec. on $(-\infty, 3)$ since $f' < 0$

f(x) is inc. on $(3, \infty)$ since $f' > 0$

4. Determine the interval(s) on which the function $f(x) = \frac{x^3}{3} + \frac{x^2}{2} - 6x + 5$ is (a) increasing and (b) decreasing.

$x = -3$ c.p.

$x = 2$ c.p.

$f'(x) = x^2 + x - 6$

$0 = (x+3)(x-2)$

$+++$ $---$ $+++$ f' at -3, 2

f(x) is inc. on $(-\infty, -3) \cup (2, \infty)$ since $f' > 0$

f(x) is dec. on $(-3, 2)$ since $f' < 0$

5. Use the Concavity Test to determine the interval(s) on which the graph of the function $f(x) = x^3 - 3x^2 - 9x + 1$ is (a) concave up and (b) concave down.

$f'(x) = 3x^2 - 6x - 9$

~~$0 = x^2 - 2x - 3$~~

~~$0 = (x-3)(x+1)$~~

~~$x = 3$ c.p. $x = -1$ c.p.~~

$f''(x) = 6x - 6$

$0 = 6x - 6$ $x = 1$

$- - -$ $+ + +$ f'' at 1

f(x) is concave down on $(-\infty, 1)$ since $f'' < 0$

f(x) is concave up on $(1, \infty)$ since $f'' > 0$

6. Find all the points of inflection of the function $f(x) = x^4 - 6x^2 + 12$

$f'(x) = 4x^3 - 12x$

$f''(x) = 12x^2 - 12$

$0 = 12x^2 - 12$

$0 = (x-1)(x+1)$

$x = 1$ $x = -1$

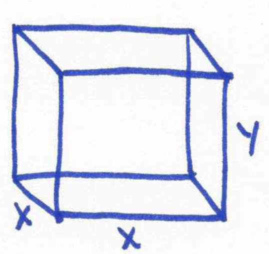
$++$ $---$ $++$ f'' at -1, 1

f(x) has Points of inflection at $x = -1$ and $x = 1$, since f'' changes sign

7. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single strand electric fence. With 800 meters of wire at your disposal, what is the largest area you can enclose, and what are the dimensions of the rectangle?

see next page (Page 17)

8. You need to build a rectangular steel tank with an open top and a square base with a volume of 500 ft^3. Your tank is to be made by welding thin stainless steel plates together along their edges. What dimensions (base width and height) would result in a tank of minimum weight?



Minimize Weight

Minimize Surface Area

$V = x^2 y$

$500 = x^2 y$

$y = \frac{500}{x^2}$

$A = x^2 + 4xy$

$A = x^2 + 4x\left(\frac{500}{x^2}\right) = x^2 + \frac{2000}{x}$

$A' = 2x - \frac{2000}{x^2}$

$0 = \frac{2x^3 - 2000}{x^2}$

$0 = 2x^3 - 2000$

$x = 10$

$- - -$ $+ + +$ A' on 0, 10

$A(10) = 100 + 200$

$= 300\ ft^2$

$x = 10$ $y = 5$

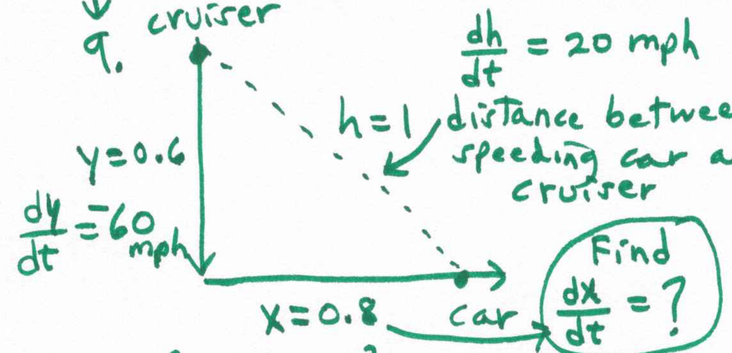
The min Area is $300\ ft^2$ at $x = 10\ ft$ since A' changes from Neg. to Pos.

9. A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving due east. When the cruiser is 0.6 miles north of the intersection and the car is 0.8 miles to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the police cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?

10. A spherical balloon is inflated with helium at the rate of $100\pi \frac{ft^3}{\text{min}}$. How fast is the balloon's radius increasing at the instant the radius is 5 feet? How fast is the surface area increasing at that instant?

11. The volume of a cube is increasing at the rate of $1200 \frac{cm^3}{\text{min}}$ at the instant its edges are 20 cm long. At what rate are the edges changing at that instant?

9.



$\frac{dh}{dt} = 20$ mph

Find $\frac{dx}{dt} = ?$

$x^2+y^2=h^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2h\frac{dh}{dt}$

$0.8\frac{dx}{dt} + 0.6(-60) = 1\cdot 20$

$0.8\frac{dx}{dt} - 36 = 20$

$0.8\frac{dx}{dt} = 56$

$\frac{dx}{dt} = 70$ mph

The speed of the car is 70 mph

10. $V = 100\pi\ ft^3/min$

$V = \frac{4}{3}\pi r^3$ $\quad r = 5$ ft

$A = 4\pi r^2$ $\quad \frac{dA}{dt} = ?$

$\frac{dV}{dt} = 3\cdot\frac{4}{3}\pi r^2 \frac{dr}{dt}$

$100\pi = 4\pi r^2 \frac{dr}{dt}$

$100\pi = 4\pi(25)\frac{dr}{dt}$

$1 = \frac{dr}{dt}$

$\frac{dr}{dt} = 1$ foot/min

$\frac{dA}{dt} = 8\pi r\frac{dr}{dt}$

$\frac{dA}{dt} = 8\pi(5)(1)$

$\frac{dA}{dt} = 40\pi\ ft^2/min$

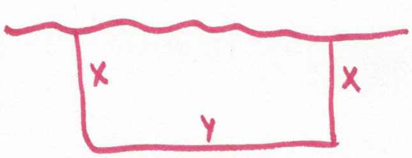
11. $V = x^3$ $\quad \frac{dV}{dt} = 3x^2\frac{dx}{dt}$

$1200 = 3(20)^2\frac{dx}{dt}$

$1\ cm/min = \frac{dx}{dt}$

the edge is increasing at a rate of 1 cm/min

7.



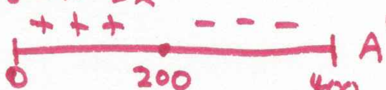
Maximize Area

$A = xy$ $\quad P = y+2x$ $\quad 800 = y+2x$

$y = 800-2x$

$A = x(800-2x) = 800x-2x^2$

$A' = 800-4x$



$0 = 800-4x$

$x = 200$

$y = 400$

A(x) is a max of 80,000 m^2 at $x = 200$ m since A' changes from pos to neg